Serial Dictatorship Mechanisms with Reservation Prices*

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Abstract

We propose a new set of mechanisms, which we call *serial dictatorship mechanisms with reservation prices* for the allocation of one indivisible good. We show that a mechanism φ satisfies *minimal tradability*, *individual rationality*, *strategy-proofness*, *consistency*, and *non wasteful tie-breaking* if and only if there exists a reservation price vector r and a priority ordering $\succ$ such that φ is a *serial dictatorship mechanism with reservation prices* based on r and $\succ$. We obtain a second characterization by replacing *individual rationality* with *non-imposition*. In both our characterizations r, $\succ$, and φ are all found simultaneously and endogenously from the properties. In addition, we show that in our model a mechanism satisfies *Pareto efficiency*, *strategy-proofness*, and *consistency* if and only if it is welfare equivalent to a classical *serial dictatorship*. Finally, we illustrate how the normative requirements governing the functioning of some real life markets and the mechanisms that these markets use are reasonably well captured by our model and results.

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1 Introduction

In many markets the resources to be allocated are overdemanded. The rationing does not always happen through market mechanisms, which leads to inefficiencies, i.e., the objects or services are typically not allocated to the agents who value them, and can pay, the most. In such markets, priority orderings (e.g., consumer queues, waiting lists, and so on) often emerge as the primary criteria for rationing the demand, with payments being only secondary. Given the private and social costs of the inherent inefficiencies, *why do such markets exist*? One possible explanation is that priority orderings may be preferred because they capture social values such as *egalitarianism* and *orderliness* (Mann, 1969). When the object or service to be allocated is some form of basic need, priority orderings may be regarded as a *just* procedure (Konow, 2003). Along the same lines, recent experiments show that agents' preferences extend beyond their own allocation and payments; the mechanism that generates the outcomes is important too, and agents also value the *procedural fairness* that comes with priority orderings (Dold and Khadjavi, 2017). In this paper, we provide *normative justifications* for the existence of such markets and we show that a set of formal normative criteria can be used to derive both, priorities over agents and individual reservation prices, while simultaneously pinning down a new mechanism (which, as we discuss in Section 8, mimics a mechanism in use in real markets) that combines agents' priorities and individual reservation prices.

Our model is as follows. There is one indivisible object and a set of potential agents. Agents' preferences over receiving the object and their own payment are represented by general utility functions that are not necessarily quasilinear. A *mechanism* allocates the object to an agent and specifies payments for all agents, i.e., it selects an *outcome*. We consider mechanisms that satisfy desirable normative criteria. Intuitively, these criteria are as follows. *Pareto efficiency* asks that the mechanism selects an outcome such that there is no other outcome that can make every agent weakly better off and at least one agent strictly better off. *Minimal tradability* requires that the object is allocated to some agent at least for some utility profile. *Individual rationality* ensures that all agents voluntarily participate. *Non-imposition* is a weakening of *individual rationality* specifying that agents who do not value the object cannot be forced to make a positive payment. *Strategy-proofness* guarantees that no agent can profitably misreport his valuation for the object. *Consistency* requires that given an outcome, if some agents who did not receive the object leave, then the outcome for all remaining agents remains the same as before. *Non wasteful tie-breaking* is a new property and requires that agents are not indifferent between [receiving the object and paying for it] and [not receiving the object and not paying anything].

In general, many fundamentally different mechanisms such as competitive mechanisms, auctions, or mechanisms that respect a priority ordering that places agents in a queue, may be compatible with some normative criteria. For instance, in our model, requiring *strategy-proofness* is compatible with a second price auction as well as with a serial dictatorship.

We first show that if a mechanism satisfies *minimal tradability*, *individual rationality*, *strategy-proofness*, and *consistency*, then there exist both a *priority ordering* $\succ$ over the set of potential agents that arranges them in a queue and a *reservation price vector* r that specifies an individual reservation price for each agent. We then introduce a *serial dictatorship mechanism with reservation prices* as follows. Agents are ranked according to an exogenously given priority order and each agent is exogenously given a reservation price that he pays if he receives the object. First, the object is offered to the agent with the highest priority. If he chooses to take it, he pays his reservation price, all other agents receive and pay nothing, and we stop. Otherwise, the object is offered to the agent with the second highest priority. If he chooses to take it, he pays his reservation price, all other agents receive and pay nothing, and we stop. Otherwise, we continue until either the object is assigned, or all (finitely many) agents have been offered the object. Note that if the reservation prices for all agents are zero, our mechanism essentially reduces to the classical *serial dictatorship mechanism*.

Our main result shows that a mechanism φ satisfies *minimal tradability*, *individual rationality*, *strategy-proofness*, *consistency*, and *non wasteful tie-breaking* if and only if there exists a reservation price vector r and a priority ordering $\succ$ such that φ is a *serial dictatorship mechanism with reservation prices* based on r and $\succ$ (Theorem 1). We obtain a second characterization by replacing *individual rationality* with *non-imposition* (Corollary 1). Note that in both our characterizations, neither the individual reservation prices nor the priority ordering the mechanism is based on are assumed as primitives; instead, the prices and priorities are derived, i.e., found endogenously, from the normative criteria, together with the serial dictatorship mechanism with reservation prices based on them. We also show that all the normative criteria that we used in our characterizations are logically independent, confirming that each criterium is indispensable (Section 7).

In addition to our main result, we show that in our model *Pareto efficiency* is a stringent requirement. First, it implies both *minimal tradability* and *individual rationality* (Lemma 1). Second, it is incompatible with individual reservation prices in the sense that requiring *Pareto efficiency* "forces" all payments to be zero (Lemma 1). Combining these results with Theorem 1, we obtain a new characterization: a mechanism φ satisfies *Pareto efficiency*, *strategy-proofness*, and *consistency* if and only if there exists a priority ordering $\succ$ such that φ is welfare equivalent to the serial dictatorship mechanism based on $\succ$ (Theorem 2). Contrasting this new characterization with our main results (Theorem 1 and Corollary 1) yields a clean normative comparison between a classical *serial dictatorship* mechanism and a more general *serial dictatorship mechanism with reservation prices*.

The remainder of this paper is organized as follows. Next, we review the related literature. In Section 2, we introduce the model and the definitions. In Section 3, we introduce the normative criteria. In Section 4, we prove a series of preliminary results that help construct (individual) reservation prices and clarify the relation between some of our properties. In Section 5, we introduce the serial dictatorship mechanism with reservation prices. In Section 6, we state and prove our characterization results. In Section 7, we show that our characterizations are tight. In Section 8, we conclude with a real life example / application that is closely related to our work.

Related Literature

While our model and our serial dictatorship mechanisms with reservation prices are new, there are several other characterizations of classical serial dictatorship mechanisms for problems of assigning objects to agents, which we refer to as *house allocation problems* (Hylland and Zeckhauser, 1979). In the following, if not otherwise specified, results refer to such problems.

Svensson (1999) shows that a mechanism is *strategy-proof*, *non-bossy*, and *neutral* if and only if it is a serial dictatorship. Ergin (2000) shows that a mechanism is *weakly Pareto efficient*, *pairwise consistent*, and *pairwise neutral* if and only if it is a serial dictatorship. For a model in which one perfectly divisible good is to be allocated among a set of agents who have single-dipped preferences, Klaus et al. (1997) first show that the entire amount of the good is assigned to one of the agents; and then, by restricting attention to situations in which the agents are not indifferent between receiving the good or not and at least one agent strictly prefers to obtain the good, they show that a mechanism is *Pareto efficient*, *strategy-proof*, and *consistent* if and only if it is a serial dictatorship. For a model in which one indivisible good needs to be allocated among a set of agents who may not necessarily want it, Pápai (2001) shows that a mechanism is *Pareto efficient* and *non-bossy* if and only if it is a serial dictatorship. For the house allocation model, Ehlers and Klaus (2007) show that if a mechanism satisfies *Pareto efficiency*, *strategy-proofness*, and *consistency*, then there exists a priority structure such that the mechanism "adapts to it." For a multiple assignment problem, Ehlers and Klaus (2003) first show that a mechanism is *Pareto efficient*, *coalitional strategy-proof*, and *resource-monotonic* if and only if it is a serial dictatorship; and then, by removing the last property, they obtain a characterization of the class of sequential dictatorship mechanisms.[1] For a model in which indivisible objects need to be allocated among agents who have responsive preferences and who each have a quota that must be filled exactly, Hatfield (2009) shows that the only *Pareto efficient*, *strategy-proof*, *non-bossy*, and *neutral* mechanisms are serial dictatorships. Besides the obvious differences in the modeling (such as for instance the absence of payments and prices in all these papers), note that with the exception of *Pareto efficiency*, *strategy-proofness*,[2] and *consistency*, the properties used in all the characterizations above are substantially different from ours. Various other notable modifications of the house allocation model and serial dictatorship mechanisms have been proposed in the literature, but these are further away from our model and from the classical serial dictatorship mechanism we relate to.[3]

Finally, our work can also be thought of as being related to the characterizations of deferred acceptance mechanisms (Kojima and Manea, 2010; Ehlers and Klaus, 2014, 2016) or immediate acceptance mechanisms (Kojima and Ünver, 2014; Doğan and Klaus, 2016); the commonality being that in those characterizations the priorities (or more generally, choice functions) are obtained from the mechanism using a set of normative criteria in the same spirit in which in our characterizations the reservation prices and priorities are derived from the mechanism using a set of properties. Furthermore, along the same lines, our results are also related to the characterizations of (single-item) second price auctions with a reserve price in which the common reserve (reservation) price is endogenously determined by various combinations of properties.[4] For instance, Sakai (2013) shows that a mechanism satisfies *weak efficiency*, *strategy-proofness*, and *non-imposition* if and only if it is either a second price auction with a reserve price or the no-trade mechanism.

[1] Unlike in a serial dictatorship, in a sequential dictatorship the order of the dictators is not exogenously given. Instead, this order is generated iteratively, where for each dictator after the first one (which is exogenously established), the current dictator's choice determines who the next dictator is.

[2] *Strategy-proofness* is a key property that is "obviously" satisfied – in the sense of Li (2016) – by all the classical serial dictatorship mechanisms and by our own serial dictatorship with reservation prices.

[3] For instance, restricted endowment inheritance mechanisms introduced by Pápai (2000) and characterized by Pápai (2000) and Ehlers et al. (2002) are essentially serial dictatorships where in each iteration, we might have either single or twin dictators. For a model where one good needs to be assigned among agents who all have single-dipped preferences, Klaus (2001a,b) provides characterizations of mechanisms that for the largest part have serial dictatorship components. Chakraborty et al. (2015) examine the efficiency properties of a serial dictatorship modified for two-sided many-to-one matching markets with interdependent valuations and imperfect information held by one side of the market. Bade (2015) characterizes a serial dictatorship mechanism for a model in which agents may privately acquire costly information about the objects that influences their valuations for it.

[4] Note that, unlike in these characterizations in which there is one common reservation price for all agents, in our model the reservation prices are idiosyncratic.

2 The Model

We consider the situation where an indivisible object can be sold or auctioned to a set of agents, but the set of agents can change. Let $\mathbb{N}$ be the set of potential agents and $\mathcal{N}$ be the set of all non-empty finite subsets of $\mathbb{N}$, $\mathcal{N} \equiv \{N \subseteq \mathbb{N} : 0 < |N| < \infty\}$.[5]

We assume that an agent $i \in \mathbb{N}$ may have to pay a *price* $p_i \geq 0$ to participate in the auction. We denote the *set of payment vectors* for a set of agents $N \in \mathcal{N}$ by

$$\mathcal{P}(N) \equiv \left\{p = (p_i)_{i\in N} : p \in \mathbb{R}^N_+\right\}.^6$$

For any set of agents $N \in \mathcal{N}$, an *allocation vector* $a = (a_i)_{i\in N} \in \{0,1\}^N$ such that $\sum_{i\in N} a_i \in \{0,1\}$ describes whether the object is allocated or not and which agent in N receives it if the object is allocated. We denote the *set of allocation vectors* for a set of agents $N \in \mathcal{N}$ by

$$\mathcal{A}(N) \equiv \left\{a = (a_i)_{i\in N} : a \in \{0,1\}^N \text{ and } \sum_{i\in N} a_i \in \{0,1\}\right\}.$$

We assume that agents only care about receiving the object or not and their own payment. Each agent $i \in \mathbb{N}$ has preferences that are: (i) strictly decreasing in the price paid; (ii) such that given the same price, receiving the object is weakly better than not receiving it; and (iii) either there exists a price which makes the agent indifferent between [receiving the object at this price] and [not receiving it and paying nothing], or he strictly prefers to [obtain the object, whatever the price] over [not receiving it and paying nothing]. Formally, we represent an agent i's preferences ($i \in \mathbb{N}$) by a utility function $u_i : \{0,1\} \times \mathbb{R}_+ \to \mathbb{R}$ that satisfies the following three properties:

(i) if $0 \leq p_i' < p_i$, then $u_i(0,p_i') > u_i(0,p_i)$ and $u_i(1,p_i') > u_i(1,p_i)$;

(ii) for each $p_i \geq 0$, $u_i(1,p_i) \geq u_i(0,p_i)$; and

(iii) either there exists a price v_i such that $u_i(1,v_i) = u_i(0,0)$, or for each $p_i \geq 0$, we have $u_i(1,p_i) > u_i(0,0)$ and $v_i \equiv \infty$; v_i is agent i's *valuation* of the indivisible object.[7]

An example of an agent i's preferences with valuation v_i are quasilinear preferences u_i defined for each $(a_i,p_i) \in \{0,1\} \times \mathbb{R}_+$ by $u_i(a_i,p_i) = v_i a_i - p_i$.

[5] A finite set of potential agents would not change any of our results.

[6] Setting the set of payment vectors equal to the Cartesian product of a discrete or finite price set would not change any of our results.

[7] Requiring continuity of u_i would be a less general assumption that guarantees the existence of valuation v_i.

We denote the *set of utility profiles* for a set of agents $N \in \mathcal{N}$ by

$$\mathcal{U}(N) \equiv \{u = (u_i)_{i \in N}: \text{ for each } i \in N,\ u_i : \{0,1\} \times \mathbb{R}_+ \to \mathbb{R} \text{ satisfies (i), (ii), and (iii)}\}$$

and the associated *set of valuation vectors* by

$$\mathcal{V}(N) \equiv \left\{v = (v_i)_{i \in N}: \text{ for each } i \in N,\ v_i \in \mathbb{R}^N \cup \{\infty\}\right\}.$$

A *problem* (of auctioning an indivisible object among a group of agents) γ is a pair $(N, u) \in \mathcal{N} \times \mathcal{U}(N)$. We denote the *set of all problems* for $N \in \mathcal{N}$ by Γ^N.

An *outcome* for any problem $\gamma \in \Gamma^N$ consists of an allocation vector $a \in \mathcal{A}(N)$ and a payment vector $p \in \mathcal{P}(N)$. We denote the *set of outcomes* for a problem $\gamma \in \Gamma^N$ by

$$\mathcal{O}(N) \equiv \mathcal{A}(N) \times \mathcal{P}(N).$$

A *mechanism* φ is a function that assigns an outcome to each problem. Formally, for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, $\varphi(\gamma) \in \mathcal{O}(N)$. Note that we can also represent a mechanism φ by its *allocation rule* α and by its *payment rule* π, i.e., for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, $\alpha : \Gamma^N \to \mathcal{A}(N)$, $\pi : \Gamma^N \to \mathcal{P}(N)$, and $\varphi(\gamma) = (\alpha(\gamma), \pi(\gamma))$. Let $\varphi_i(\gamma) = (\alpha_i(\gamma), \pi_i(\gamma))$ denote the *allotment* of agent i at outcome $\varphi(\gamma)$.

Given $N \in \mathcal{N}$, a vector $x \in \mathbb{R}^N$, and $M \subseteq N$, let x_M denote the vector $(x_i)_{i \in M} \in \mathbb{R}^M$. It is the restriction of vector x to the subset of agents M. We also use the notation $x_{-i} = x_{N \setminus \{i\}}$. For example, $(\bar{x}_i, x_{-i})$ denotes the vector obtained from x by replacing x_i with $\bar{x}_i$. We use corresponding notational conventions for utility profiles.

3 Properties of Mechanisms

Our first property is *Pareto efficiency*. An outcome is *Pareto efficient* if there is no other outcome that makes every agent weakly better off and at least one agent strictly better off.

For $N \in \mathcal{N}$, an outcome $(a, p) \in \mathcal{O}(N)$ is *Pareto efficient* for utility profile $u \in \mathcal{U}(N)$ if there is no outcome $(a', p') \in \mathcal{O}(N)$ such that for each $i \in N$, $u_i(a'_i, p'_i) \geq u_i(a_i, p_i)$, and for some $j \in N$, $u_j(a'_j, p'_j) > u_j(a_j, p_j)$.

Pareto Efficiency: A mechanism φ satisfies *Pareto efficiency* if for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, $\varphi(\gamma)$ is a *Pareto efficient* outcome.

In our setting, *Pareto efficiency* is a stringent requirement; in particular, as we show in Lemma 1 in the next section, it implies our next three properties.

The first of these properties is due to Sakai (2013) and it ensures that for each group of agents, at least for some utility profile the object is allocated to some agent.

Minimal Tradability: A mechanism φ satisfies *minimal tradability* if for each $N \in \mathcal{N}$, there exists $u \in \mathcal{U}(N)$ and $i \in N$ such that $\alpha_i(N, u) = 1$.

The following property allows agents who do not value the object to withdraw from the problem at no cost (i.e., these agents cannot be forced to pay a positive price for the object).

Non-Imposition: A mechanism φ satisfies *non-imposition* if for each $N \in \mathcal{N}$, each $\gamma \in \Gamma^N$, and each $i \in N$, if u_i is such that $v_i = 0$, then $\pi_i(\gamma) = 0$.

Non-imposition was first introduced by Sakai (2008) who also observed that this property is very weak as it is satisfied by virtually all of the auction mechanisms in the literature.

For $N \in \mathcal{N}$, an outcome $(a, p) \in \mathcal{O}(N)$ is *individually rational* for utility profile $u \in \mathcal{U}(N)$ if for each $i \in N$, $u_i(a_i, p_i) \geq u_i(0, 0)$. Equivalently, an outcome $(a, p) \in \mathcal{O}(N)$ is individually rational for utility profile $u \in \mathcal{U}(N)$ with associated valuation vector $v \in \mathcal{V}(N)$ if for each $i \in N$, **(IR1)** $[a_i = 0$ implies $p_i = 0]$ and **(IR2)** $[a_i = 1$ implies $p_i \leq v_i]$. By requiring the mechanism to only choose individually rational outcomes we express the idea of voluntary participation.

Individual Rationality: A mechanism φ satisfies *individual rationality* if for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, $\varphi(\gamma)$ is an individually rational outcome.

Remark 1. If a mechanism φ satisfies *individual rationality*, then it satisfies *non-imposition*.

Strategy-proofness requires that no agent can ever benefit from misrepresenting his preferences.

Strategy-Proofness: A mechanism φ satisfies *strategy-proofness* if for each $N \in \mathcal{N}$, each $(N, u) \in \Gamma^N$, each $i \in N$, and each u'_i such that $u' \equiv (u'_i, u_{-i}) \in \mathcal{U}(N)$, $u_i(\varphi_i(N, u)) \geq u_i(\varphi_i(N, u'))$.

That is, a mechanism is *strategy-proof* if (in the associated direct revelation game) it is a weakly dominant strategy for each agent to report his valuation truthfully.

Consistency, first introduced by Thomson (1983), is one of the key properties in many frameworks with variable populations.[8] Adapted to our setting, *consistency* requires that if

[8]Thomson (2015) provides an extensive survey of *consistency* in various applications.

some agents who did not receive the object leave, then the allocation and the payments for all remaining agents should not change. Let $N \in \mathcal{N}$, $\gamma = (N, u) \in \Gamma^N$, and $M \subseteq N$. Then, $\gamma_M = (M, u_M)$.

Consistency: A mechanism φ satisfies *consistency* if for each $N \in \mathcal{N}$, each $\gamma \in \Gamma^N$, and each $M \subseteq N$, $\sum_{i \in N \setminus M} \alpha_i(\gamma) = 0$ implies that $\varphi(\gamma_M) = \varphi(\gamma)_M$.

Our last property is new and excludes that the mechanism selects outcomes where the agent who receives the object is indifferent between his allotment and not receiving the object at price zero. The idea behind this property is to not wastefully assign the object to such an agent because another agent might prefer to receive it. In that sense, *non-wasteful tie-breaking* is a mild efficiency requirement.

Non Wasteful Tie-Breaking: A mechanism φ satisfies *non wasteful tie-breaking* if for each $N \in \mathcal{N}$, each $\gamma \in \Gamma^N$, and each $i \in N$, $\alpha_i(\gamma) = 1$ implies that $u_i(1, \pi_i(\gamma)) \neq u_i(0, 0)$.

4 Preliminary Results: Relations Among the Properties and (Individual) Reservation Prices

We first show that for any problem, if a mechanism satisfies *Pareto efficiency*, then all agents make zero payments and, unless all agents have zero valuations, the object is always allocated. Furthermore, *Pareto efficiency* implies *minimal tradability*, *non-imposition*, and *individual rationality*.

Lemma 1. *Assume that mechanism φ satisfies* Pareto efficiency. *Then,*

(i) *for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, we have that for all $i \in N$, $\pi_i(\gamma) = 0$ and*

(ii) *if the associated valuation vector $v \in \mathcal{V}(N)$ is such that for some $j \in N$, $v_j > 0$, then $\sum_{k \in N} \alpha_k(\gamma) = 1$.*

Furthermore, φ satisfies

(iii) *minimal tradability,*

(iv) *non-imposition, and*

(v) *individual rationality.*

Proof. Assume that mechanism φ satisfies *Pareto efficiency*.

(i) Let $N \in \mathcal{N}$ and $\gamma \in \Gamma^N$. Assume, by contradiction, $(\alpha(\gamma), \pi(\gamma))$ is such that for some $i \in N$, $\pi_i(\gamma) > 0$. Let $(a, p) \in \mathcal{O}(N)$ be such that $a = \alpha(\gamma)$, $p_i = 0$, and $p_{-i} = \pi_{-i}(\gamma)$. Then, by property (i) of utility function u_i we have that $u_i(\alpha_i(\gamma), \pi_i(\gamma)) < u_i(a_i, p_i)$ and for all $j \in N \setminus \{i\}$, $u_j(\alpha_j(\gamma), \pi_j(\gamma)) = u_j(a_j, p_j)$. Hence, (a, p) is a *Pareto improvement* over outcome $(\alpha(\gamma), \pi(\gamma))$, contradicting *Pareto efficiency* of φ.

(ii) Let $N \in \mathcal{N}$ and $\gamma \in \Gamma^N$ with an associated valuation vector $v \in \mathcal{V}(N)$ such that for some $j \in N$, $v_j > 0$. Assume, by contradiction, $\sum_{k \in N} \alpha_k(\gamma) = 0$. Let $(a, p) \in \mathcal{O}(N)$ be such that $a_j = 1$ and by (i) above, $p = \pi(\gamma) = 0$. Then, by properties (i) and (iii) of utility function u_j we have that $u_j(\alpha_j(\gamma), \pi_j(\gamma)) = u_j(0, 0) = u_j(1, v_j) < u_j(1, 0) = u_j(a_j, p_j)$. Furthermore, for all $k \in N \setminus \{j\}$, $u_k(\alpha_k(\gamma), \pi_k(\gamma)) = u_k(a_k, p_k)$. Hence, (a, p) is a *Pareto improvement* over outcome $(\alpha(\gamma), \pi(\gamma))$, contradicting *Pareto efficiency* of φ.

(iii) To see that φ satisfies *minimal tradability*, observe that by (ii), for any $N \in \mathcal{N}$ and $\gamma \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$ such that for some $j \in N$, $v_j > 0$, we have $\sum_{k \in N} \alpha_k(\gamma) = 1$.

(iv) Since by (i) for each $N \in \mathcal{N}$, each $\gamma \in \Gamma^N$, and each $i \in N$, $\pi_i(\gamma) = 0$. Hence, *non-imposition* is vacuously satisfied.

(v) To see that φ satisfies *individual rationality*, observe that by (i), for each $i \in N$, $\alpha_i(\gamma) = 0$ implies $\pi_i(\gamma) = 0$ (IR1), and $\alpha_i(\gamma) = 1$ implies $\pi_i(\gamma) = 0 \leq v_i$ (IR2). □

Our next result is a generalization of a result for quasi-linear utility functions due to (Sakai, 2013, Proposition 1 (iii)).

Lemma 2. *Assume that mechanism φ satisfies strategy-proofness and non-imposition. Then, φ satisfies individual rationality.*

Proof. Assume that φ satisfies *strategy-proofness* and *non-imposition*. Let $N \in \mathcal{N}$, $(N, u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$, $i \in N$, and $u' = (u'_i, u_{-i}) \in \mathcal{U}(N)$ with associated valuation vector $v' = (0, v_{-i}) \in \mathcal{V}(N)$.

(IR1) Suppose that $\alpha_i(N, u) = 0$ and, in contradiction to (IR1), $\pi_i(N, u) > 0$. By property (i) of utility function u_i, $u_i(\varphi_i(N, u)) = u_i(0, \pi_i(N, u)) < u_i(0, 0)$. By property (ii) of utility function u_i, $u_i(0, 0) \leq u_i(1, 0)$.

By *non-imposition*, we have $\pi_i(N, u') = 0$. Hence, $\varphi_i(N, u') \in \{(0, 0), (1, 0)\}$ and $u_i(\varphi_i(N, u)) < u_i(\varphi_i(N, u'))$, contradicting *strategy-proofness*. Thus, $\alpha_i(N, u) = 0$ implies $\pi_i(N, u) = 0$.

(IR2) Suppose that $\alpha_i(N,u) = 1$ and, in contradiction to (IR2), $\pi_i(N,u) > v_i$ (≥ 0). By property (iii) of utility function u_i, $v_i \neq \infty$ and $u_i(1,v_i) = u_i(0,0)$. By property (i) of utility function u_i, $u_i(\varphi_i(N,u)) = u_i(1,\pi_i(N,u)) < u_i(1,v_i) = u_i(0,0)$. By property (ii) of utility function u_i, $u_i(0,0) \leq u_i(1,0)$.

By *non-imposition*, we have $\pi_i(N,u') = 0$. Hence, $\varphi_i(N,u') \in \{(0,0),(1,0)\}$ and $u_i(\varphi_i(N,u)) < u_i(\varphi_i(N,u'))$, contradicting *strategy-proofness*. Thus, $\alpha_i(N,u) = 1$ implies $\pi_i(N,u) \leq v_i$. □

Combining Lemma 2 and Remark 1, we make the following observation.

Remark 2. Under *strategy-proofness*, *non-imposition* and *individual rationality* are equivalent.

Next, we show the existence of a reservation price vector that specifies individual reservation prices for each agent. Assume that mechanism φ satisfies *strategy-proofness*. Then, for each $i \in \mathbb{N}$, we define an *individual reservation price* $r_i \geq 0$ as follows.

Let $N = \{i\}$. Define the *price range* of mechanism φ for agent i with preferences u_i as the set of all possible prices at which he could obtain the object, i.e.,

$$P_i^{\varphi} = \{p_i \in \mathbb{R}_+ : \varphi_i(\{i\},u_i) = (1,p_i) \text{ for some } u_i \in \mathcal{U}(\{i\})\}.$$

We show that $|P_i|^{\varphi} \leq 1$.

Suppose that $|P_i^{\varphi}| > 1$. Then, there exist $p_i, p_i' \in P_i^{\varphi}$ and, without loss of generality, assume $p_i > p_i'$. Hence, there exist utility functions $u_i, u_i' \in \mathcal{U}(\{i\})$ such that $\varphi_i(\{i\},u_i) = (1,p_i)$ and $\varphi_i(\{i\},u_i') = (1,p_i')$. Then, agent i with preferences represented by u_i can receive the object at the lower price p_i' if he pretends his preferences are represented by u_i'. Thus, in contradiction to *strategy-proofness*, by property (i) of utility function u_i, $u_i(\varphi_i(\{i\},u_i')) = u_i(1,p_i') > u_i(1,p_i) = u_i(\varphi_i(\{i\},u_i))$.

If $P_i^{\varphi} \equiv \emptyset$, then we set $r_i = \infty$. Otherwise, r_i is defined via $P_i^{\varphi} = \{r_i\}$.

Recall that by definition, for each $i \in \mathbb{N}$, we have that for the valuation v_i associated with u_i, $v_i \geq 0$. For each $i \in \mathbb{N}$, *minimal tradability* implies that $r_i < \infty$ and *individual rationality* (IR2) implies that $v_i \geq r_i$. Note that at $v_i = r_i < \infty$, agent i is indifferent between [not receiving the object and not paying anything] and [receiving the object and paying $r_i = v_i$]. We have established the following result.

Lemma 3. *Assume that mechanism φ satisfies* minimal tradability, individual rationality, and strategy-proofness. *Then, for each agent $i \in \mathbb{N}$, there exists an individual reservation price $r_i \geq 0$ such that for each utility function $u_i \in \mathcal{U}(\{i\})$ with associated valuation $v_i \in \mathcal{V}(\{i\})$:*

(i) *$v_i > r_i$ implies $\varphi_i(\{i\}, u_i) = (1, r_i)$,*

(ii) *$v_i = r_i$ implies $\varphi_i(\{i\}, u_i) \in \{(0,0), (1, r_i)\}$, and*

(iii) *$v_i < r_i$ implies $\varphi_i(\{i\}, u_i) = (0, 0)$.*

With our next lemma, we show that for any problem, if an agent receives the object, then his valuation has to be weakly larger than his individual reservation price (which also equals his payment); otherwise, his payment is necessarily null.

Lemma 4. *Assume that mechanism φ satisfies* minimal tradability, individual rationality, strategy-proofness, *and* consistency. *Then, for each $N \in \mathcal{N}$, each $\gamma \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$, and each $i \in N$,*

(i) *if $\alpha_i(\gamma) = 1$, then $\pi_i(\gamma) = r_i \leq v_i$ (with r_i as in Lemma 3) and*

(ii) *if $\alpha_i(\gamma) = 0$, then $\pi_i(\gamma) = 0$.*

Proof. Assume that φ satisfies all the properties in the lemma. Let $N \in \mathcal{N}$, $\gamma \in \Gamma^N$, and $i \in N$.

(i) Assume that $\alpha_i(\gamma) = 1$. By *individual rationality* (IR2), $\pi_i(\gamma) \leq v_i$. By *consistency* and *Lemma 3*, $(\alpha_i(\gamma), \pi_i(\gamma)) = \varphi_i(\{i\}, u_i) = (1, r_i)$. Hence, $\pi_i(\gamma) = r_i \leq v_i$.

(ii) If $\alpha_i(\gamma) = 0$, then by *individual rationality* (IR1), $\pi_i(\gamma) = 0$. □

5 Serial Dictatorship with Reservation Prices

In order to define a serial dictatorship with reservation prices we first need to fix a *priority ordering* and *reservation prices.*

A *priority ordering* $\succ$ over the set of potential agents $\mathbb{N}$ is a complete, asymmetric, and transitive binary relation, with the interpretation that for any two distinct agents $i, j \in \mathbb{N}$, $i \succ j$ means that i has a higher priority than j. Let Ω denote the *set of all priority orderings* over $\mathbb{N}$.

We assume that for each agent $i \in \mathbb{N}$ a *(fixed) reservation price* $f_i \geq 0$ exists. We interpret f_i as the smallest price at which the object can be allocated to agent i. We denote a *vector of (fixed) reservation prices* for the set of potential agents $\mathbb{N}$ by $f = (f_i)_{i \in \mathbb{N}}$ and by $\mathcal{F}$ we denote the *set of all (fixed) reservation price vectors* for $\mathbb{N}$.[9]

Given a reservation price vector $f \in \mathcal{F}$ and a priority ordering $\succ \in \Omega$, the *serial dictatorship mechanism with reservation prices* based on f and $\succ$ is denoted by $\psi^{(f,\succ)}$ and determines an outcome for each problem $\gamma = (N, u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$ as follows.

> **First**, the agent with the highest priority in N is considered. Let $i \in N$ be this agent. If $v_i > f_i$, then agent i obtains the object and pays f_i. All remaining agents receive and pay nothing. If $v_i \leq f_i$, then agent i receives and pays nothing (and we continue).
>
> **Second (third, etc.)**, the agent with the next highest priority in N is considered. Let $j \in N$ be this agent. If $v_j > f_j$, then agent j obtains the object and pays f_j. All remaining agents receive and pay nothing. If $v_j \leq f_j$, then agent j receives and pays nothing (and we continue until either an agent's value exceeds his reservation price, or all of the finitely many agents have been considered).

Formally, for each $N \in \mathcal{N}$ and each $\gamma = (N, u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$, define the set of agents who have a larger valuation than their reservation price by

$$U^f(\gamma) \equiv \{i \in N : v_i > f_i\}.$$

The serial dictatorship mechanism with reservation prices $\psi^{(f,\succ)}$ assigns the uniquely determined outcome $\psi^{(f,\succ)}(\gamma) \in \mathcal{O}(N)$ such that:

(a) if $i \in N \setminus U^f(\gamma)$, then $\psi_i^{(f,\succ)}(\gamma) = (0, 0)$;

(b) if $i \in U^f(\gamma)$ and there exists $j \in U^f(\gamma)$ such that $j \succ i$, then $\psi_i^{(f,\succ)}(\gamma) = (0, 0)$; and

(c) if $j \in U^f(\gamma)$ and for each $i \in U^f(\gamma) \setminus \{j\}$, $j \succ i$, then $\psi_j^{(f,\succ)}(\gamma) = (1, f_j)$.

Note that if the reservation prices are zero for all agents, we obtain the classical serial dictatorship mechanism. That is, given the reservation price vector $\mathbf{0} = (0, 0, \dots) \in \mathcal{F}$ and a priority ordering $\succ \in \Omega$, $\psi^{(\mathbf{0},\succ)}$ is a *serial dictatorship mechanism.*

[9] Note that for each agent, the reservation price f_i is fixed as part of the definition of a serial dictatorship mechanism with reservation prices and hence it is exogenously given, whereas the individual reservation price r_i in Lemmas 3 and 4 is derived endogenously from a set of normative properties satisfied by a mechanism; r_i is guaranteed to exist only for mechanisms that satisfy those properties.

6 Main Results: Characterizations

Theorem 1. *A mechanism φ satisfies minimal tradability, individual rationality, strategy-proofness, consistency, and non wasteful tie-breaking if and only if there exist a reservation price vector $r \in \mathcal{F}$ and a priority ordering $\succ \in \Omega$ such that φ is a serial dictatorship mechanism with reservation prices based on r and $\succ$, i.e., $\varphi = \psi^{(r,\succ)}$.*

Proof. It is easy to see that any serial dictatorship mechanism with reservation prices induced by some reservation price vector $f \in \mathcal{F}$ and some priority ordering $\succ \in \Omega$ satisfies all the properties in the theorem.

To prove the uniqueness part, assume that φ satisfies all the properties in the theorem. Our proof proceeds in three steps. First, we fix the individual reservation price vector $r \in \mathcal{F}$ defined in Lemma 3. Second, we define a priority ordering $\succ$ over $\mathbb{N}$. Third, we prove that $\varphi = \psi^{(r,\succ)}$.

Step 1. Construction of reservation price vector $r \in \mathcal{F}$.

Since mechanism φ satisfies *minimal tradability*, *individual rationality*, and *strategy-proofness*, by Lemma 3, we can define the individual reservation price vector $r \in \mathcal{F}$ such that for each agent $i \in \mathbb{N}$, $r_i \geq 0$ and such that for each utility function $u_i \in \mathcal{U}(\{i\})$ with associated valuation $v_i \in \mathcal{V}(\{i\})$:

(i) $v_i > r_i$ implies $\varphi_i(\{i\}, u_i) = (1, r_i)$,

(ii) $v_i = r_i$ implies $\varphi_i(\{i\}, u_i) \in \{(0,0), (1, r_i)\}$, and

(iii) $v_i < r_i$ implies $\varphi_i(\{i\}, u_i) = (0,0)$.

Step 2. Construction of priority order $\succ \in \Omega$ over $\mathbb{N}$.

Let $i, j \in \mathbb{N}$, $i \neq j$. By *minimal tradability*, there exists $u = (u_i, u_j) \in \mathcal{U}(\{i,j\})$ such that for $k \in \{i,j\} \equiv \{k,l\}$, $\alpha_k(\{k,l\}, u) = 1$. By *consistency* and Lemma 3 (see (i) and (ii) above), $\varphi_k(\{k,l\}, u) = \varphi_k(\{k\}, u_k) = (1, r_k)$.

Let $u' = (\bar{u}_k, u_l) \in \mathcal{U}(\{k,l\})$ with associated valuation vector $v' = (r_k + 1, v_l) \in \mathcal{V}(\{k,l\})$. Then, by *strategy-proofness*, $\alpha_k(\{k,l\}, u') = 1$ (in fact, we even have $\varphi_k(\{k,l\}, u') = (1, r_k)$)).

Let $(\bar{u}_k, \bar{u}_l) \in \mathcal{U}(\{k,l\})$ with associated valuation vector $(r_k + 1, r_l + 1) \in \mathcal{V}(\{k,l\})$. By *consistency*, the object continues to remain allocated at problem $(\{k,l\}, (\bar{u}_k, \bar{u}_l))$. To see this, observe that otherwise, if the object is not allocated anymore, starting from $(\{k,l\}, (\bar{u}_k, \bar{u}_l))$ and removing agent l, by *consistency* we would have $\alpha_k(\{k\}, \bar{u}_k) = 0$, which would contradict that $\varphi_k(\{k\}, \bar{u}_k) = (1, r_k)$ (by (i)). Thus, one of the agents in $\{i, j\} \equiv \{k, l\}$ receives the object. If $\alpha_i(\{i, j\}, (\bar{u}_i, \bar{u}_j)) = 1$, then set $i \succ j$. Otherwise, if $\alpha_j(\{i, j\}, (\bar{u}_i, \bar{u}_j)) = 1$, then set $j \succ i$.

We now prove the transitivity of $\succ$. Assume, by contradiction, that there exist distinct agents $i, j, k \in \mathbb{N}$ such that $i \succ j$, $j \succ k$, and $k \succ i$. Assume that for any of these agents $a \in \{i, j, k\}$, $\bar{u}_a$ is the utility function used to determine $\succ$ with associated valuation $r_a + 1$. Hence, $\alpha_i(\{i, j\}, (\bar{u}_i, \bar{u}_j)) = 1$, $\alpha_j(\{j, k\}, (\bar{u}_j, \bar{u}_k)) = 1$, and $\alpha_k(\{i, k\}, (\bar{u}_i, \bar{u}_k)) = 1$.

By *minimal tradability*, there exists $u = (u_i, u_j, u_k) \in \mathcal{U}(\{i, j, k\})$ such that for $l \in \{i, j, k\} \equiv \{l, m, n\}$, $\alpha_l(\{l, m, n\}, v) = 1$. By *consistency* and Lemma 3 (see (i) and (ii) above), $\varphi_l(\{l, m, n\}, u) = \varphi_l(\{l\}, u_l) = (1, r_l)$.

Let $u' = (\bar{u}_l, u_m, u_n) \in \mathcal{U}(\{l, m, n\})$ with associated valuation vector $v' = (r_l + 1, v_m, v_n) \in \mathcal{V}(\{l, m, n\})$. Then, by *strategy-proofness*, $\alpha_l(\{l, m, n\}, u') = 1$ (in fact, we even have $\varphi_l(\{l, m, n\}, u') = (1, r_l)$)).

Let $u'' = (\bar{u}_l, \bar{u}_m, u_n) \in \mathcal{U}(\{l, m, n\})$ with associated valuation vector $v'' = (r_l + 1, r_m + 1, v_n) \in \mathcal{V}(\{l, m, n\})$. By *consistency*, the object continues to remain allocated at problem $(\{l, m, n\}, u'')$. To see this, observe that otherwise, if the object is not allocated anymore, starting from $(\{l, m, n\}, u'')$ and removing agent n, by *consistency* we would have $\alpha_l(\{l, m\}, (\bar{u}_l, \bar{u}_m) = 0$ and $\alpha_m(\{l, m\}, (\bar{u}_l, \bar{u}_m) = 0$, which would contradict that either $l \succ m$ or $m \succ l$. Thus, one of the agents in $\{i, j, k\} \equiv \{l, m, n\}$ receives the object.

Let $(\bar{u}_l, \bar{u}_m, \bar{u}_n) \in \mathcal{U}(\{l, m, n\})$ with associated valuation vector $(r_l + 1, r_m + 1, r_n + 1) \in \mathcal{V}(\{l, m, n\})$. By *consistency* (if agent n did not receive the object before) or by *strategy-proofness* (if agent n did receive the object before), one of the agents in $\{i, j, k\} \equiv \{l, m, n\}$ receives the object, without loss of generality, agent i, i.e., $\alpha_i(\{i, j, k\}, (\bar{u}_i, \bar{u}_j, \bar{u}_k)) = 1$. By *consistency*, $\alpha_i(\{i, k\}, (\bar{u}_i, \bar{u}_k)) = 1$, contradicting $k \succ i$ (and hence, $\alpha_k(\{i, k\}, (\bar{u}_i, \bar{u}_k)) = 1$).

Step 3. Proof that $\varphi = \psi^{(r, \succ)}$.

We show that φ always assigns the object and payments as if it is a serial dictatorship mechanism based on $r \in \mathcal{F}$ and $\succ \in \Omega$, i.e., we show that for each $N \in \mathcal{N}$, each $\gamma = (N, u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$, and $U^r(\gamma) \equiv \{i \in N : v_i > r_i\}$, φ assigns the following uniquely determined outcome:

(a) if $i \in N \setminus U^r(\gamma)$, then $\varphi_i(\gamma) = (0, 0)$;

(b) if $i \in U^r(\gamma)$ and there exists $j \in U^r(\gamma)$ such that $j \succ i$, then $\varphi_i(\gamma) = (0, 0)$; and

(c) if $j \in U^r(\gamma)$ and for each $i \in U^r(\gamma) \setminus \{j\}$, $j \succ i$, then $\varphi_j(\gamma) = (1, r_j)$.

Recall that by *individual rationality* (IR1), if $i \in N$ and $\alpha_i(\gamma) = 0$, then $\pi_i(\gamma) = 0$. Furthermore, by Lemma 4 (i), if $i \in N$ and $\alpha_i(\gamma) = 1$, then $\pi_i(\gamma) = r_i$. Hence, we only need to prove that the allocation rule $\alpha = \alpha^{(r,\succ)}$. We proceed by contradiction, considering a different object allocation in each of the Cases (a), (b), and (c) above.

Case (a): there exists $i \in N \setminus U^r(\gamma)$ such that $\alpha_i(\gamma) = 1$.

By Lemma 4 (i), $\pi_i(\gamma) = r_i$ and by $i \in N \setminus U^r(\gamma)$, we have $v_i \leq r_i$. If $v_i < \pi_i(\gamma)$, then *individual rationality* (IR2) is violated. If $v_i = \pi_i(\gamma)$, then *non wasteful tie-breaking* is violated.

Case (b): there exist $i, j \in U^r(\gamma)$ such that $j \succ i$ and $\alpha_i(\gamma) = 1$.

Assume that $(\bar{u}_i, \bar{u}_j)$ is the utility profile used to determine $j \succ i$ with associated valuation vector $(r_i + 1, r_j + 1)$. Hence, $\varphi_j(\{i, j\}, (\bar{u}_i, \bar{u}_j)) = (1, r_j)$.

Starting from problem (N, u), by *consistency* and *Lemma 3*, $\varphi_i(\{i, j\}, (u_i, u_j)) = (1, r_i)$. By *strategy-proofness*, $\alpha_i(\{i, j\}, (\bar{u}_i, u_j)) = 1$. Hence, $\alpha_j(\{i, j\}, (\bar{u}_i, u_j)) = 0$ and by Lemma 4 (ii), $\varphi_j(\{i, j\}, (\bar{u}_i, u_j)) = (0, 0)$.

Since $j \in U^r(\gamma)$, we have $v_j > r_j$. Then, in contradiction to *strategy-proofness*, we have that $u_j(\varphi_j(\{i, j\}, (\bar{u}_i, \bar{u}_j))) = u_j(1, r_j) > u_j(0, 0) = u_j(\varphi_j(\{i, j\}, (\bar{u}_i, u_j)))$ (agent j with utility function u_j and valuation v_j will beneficially misreport utility function $\bar{u}_j$ with valuation $r_j + 1$).

Case (c): for $j \in U^r(\gamma)$ such that for each $i \in U^r(\gamma) \setminus \{j\}$, $j \succ i$, we have $\alpha_j(\gamma) = 0$.

The contradictions obtained for Cases (a) and (b) above imply that for each $i \in N \setminus \{j\}$, $\alpha_i(\gamma) = 0$. If now also $\alpha_j(\gamma) = 0$, then the object is not allocated. By *consistency*, starting from problem $\gamma = (N, u)$ and removing all agents but j, we obtain $\alpha_j(\{j\}, u_j) = 0$. However, since $j \in U^r(\gamma)$, we have $v_j > r_j$, which by the definition of r_j in Lemma 3 (see (i) above) implies that $\alpha_j(\{j\}, u_j) = 1$, a contradiction. □

Lemma 2 and Theorem 1 imply the following corollary.

Corollary 1. *A mechanism φ satisfies minimal tradability, non-imposition, strategy-proofness, consistency, and non wasteful tie-breaking if and only if there exist a reservation price vector $r \in \mathcal{F}$ and a priority ordering $\succ \in \Omega$ such that φ is a serial dictatorship mechanism with reservation prices based on r and $\succ$, i.e., $\varphi = \psi^{(r,\succ)}$.*

As we discussed in the introduction, *Pareto efficiency* has been used in a number of characterizations of the classical serial dictatorship mechanism. In all these characterizations, the preferences of the agents are given by linear orders and there are no prices. Consequently, *Pareto efficiency* requires only the allocation to be efficient.[10] In contrast, in our model the preferences of the agents are given by utilities over outcomes. Consequently, *Pareto efficiency* requires both allocation and prices to be simultaneously efficient. Clearly, the latter *Pareto efficiency* requirement is more stringent than the former. But does *Pareto efficiency* over outcomes imply that the agent with the highest valuation for the object always obtains it, thus ruling out priority orderings in general? Our next result answers this question in the negative; furthermore, dropping *non-wasteful tie-breaking* and replacing *minimal tradability* together with *individual rationality* in Theorem 1 (or together with *non-imposition* in Corollary 1) by *Pareto efficiency* essentially pins down the classical serial dictatorship mechanism.

Theorem 2. *A mechanism φ satisfies* Pareto efficiency, strategy-proofness, *and* consistency *if and only if there exists a priority ordering $\succ \in \Omega$ such that φ is welfare equivalent to the serial dictatorship mechanism with reservation prices based on $\mathbf{0} \in \mathcal{F}$ and $\succ \in \Omega$, i.e., $\varphi \sim \psi^{(\mathbf{0},\succ)}$. More precisely, let $N \in \mathcal{N}$ and $\gamma \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$; then, if $v \neq (0,\ldots,0)$, $\varphi(\gamma) = \psi^{(\mathbf{0},\succ)}(\gamma)$ and if $v = (0,\ldots,0)$, $\varphi(\gamma) \in \{\psi^{(\mathbf{0},\succ)}(\gamma)\} \cup \{(a,(0,\ldots,0)) : a \in \mathcal{A}(N)\}$.*

***Proof*.** Assume that φ satisfies all the properties in the theorem. By Lemma 1 (iii), (iv), and (v), φ satisfies *minimal tradability*, *non-imposition*, and *individual rationality*. The proof is now analogous to the proof of Theorem 1, except for the following two modifications.

First, *Step 1* is no longer necessary. Instead, Lemma 1 (i) directly implies $r = \mathbf{0}$ such that for each utility function $u_i \in \mathcal{U}(\{i\})$ with associated valuation $v_i \in \mathcal{V}(\{i\})$:

(i) $v_i > 0$ implies $\varphi_i(\{i\}, u_i) = (1, 0)$ and

(ii) $v_i = 0$ implies $\varphi_i(\{i\}, u_i) \in \{(0,0),(1,0)\}$.

Second, we adjust *Step 3* as follows (note that this is the only step using *non wasteful tie-breaking*). Let $N \in \mathcal{N}$, $\gamma = (N, u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$, and $U^{\mathbf{0}}(\gamma) \equiv \{i \in N : v_i > 0\}$.

If $v = (0,\ldots,0)$, then by (ii), for all $i \in N$, $u_i(\varphi(\gamma)) = u_i(0,0) = u_i(1,0)$. Hence, for all $i \in N$, $u_i(\varphi(\gamma)) = u_i(\psi^{(\mathbf{0},\succ)}(\gamma))$.

If $v \neq \psi^{(\mathbf{0},\succ)}(\gamma)$, then for some $j \in N$, $v_j > 0$. We now adjust the arguments of *Case (a)* to obtain a contradiction.

[10] That is, in the absence of prices, *Pareto efficiency* requires that the object is not "inefficiently" allocated to an agent i who does not want it if there is another agent j who does want it; taking the object from i and giving it to j would be a *Pareto improvement*.

Case (a): there exists $i \in N \setminus U^{\mathbf{0}}(\gamma)$ such that $\varphi_i(\gamma) = (1, 0)$.

Let $(a, p) \in \mathcal{O}(N)$ be such that $a_j = 1$ and $p = \pi(\gamma) = 0$. Then, by properties (i) and (iii) of utility functions u_i and u_j we have that $u_i(\varphi(\gamma)) = u_i(1, 0) = u_i(0, 0) = u_i(a_i, p_i)$ and $u_j(\varphi(\gamma)) = u_j(0, 0) = u_j(1, v_j) < u_j(1, 0) = u_j(a_j, p_j)$. Furthermore, for all $k \in N \setminus \{i, j\}$, $u_k(\varphi_k(\gamma)) = u_k(a_k, p_k)$. Hence, (a, p) is a *Pareto improvement* over outcome $\varphi(\gamma)$, contradicting *Pareto efficiency* of φ.

The proof for *Cases (b)* and *(c)* remains unchanged and we obtain that if $v \neq \mathbf{0}$, then $\varphi(\gamma) = \psi^{(\mathbf{0},\succ)}(\gamma)$. □

While Theorem 2 is new, this result is in line with the main findings in Klaus et al. (1997) and Ehlers and Klaus (2007). In our model, it is particularly valuable because contrasting Theorem 2 with Theorem 1 and Corollary 1 yields a clean normative comparison between the classical serial dictatorship mechanism and our newly introduced serial dictatorship with reservation prices. To see this, observe that *Pareto efficiency* rules out individual reservation prices (Lemma 1 (i)) but stops short of requiring that the agent with the highest valuation for the object obtains it; instead, after "forcing" all individual reservation prices to zero, *Pareto efficiency* over outcomes remains compatible with priority orderings and essentially reduces to *Pareto efficiency* over allocations (Theorem 2). Furthermore, observe that weakening *Pareto efficiency* to *minimal tradability* and *individual rationality* (Lemma 1 (iii) and (v)) is necessary in order to allow for the possibility of having individual reservation prices, and it is also sufficient for pinning down these prices (Lemmas 3 and 4). Taken together, these observations and Theorems 1 and 2 show that considering a serial dictatorship versus a serial dictatorship with reservation prices has a normative counterpart in considering *Pareto efficiency* versus *minimal tradability* and *individual rationality*.

7 Independence of Properties

In this section, we show that our characterizations are tight. The following examples present mechanisms that satisfy all the properties in Theorem 1, Corollary 1, and Theorem 2, except for the one(s) in the title of the example.

Example 1 (***Minimal Tradability, Pareto Efficiency***)
Let φ^{nt} be the constant no-trade mechanism according to which the object is never allocated and no payments are ever made, i.e., for each $N \in \mathcal{N}$, each $\gamma \in \Gamma^N$, and each $i \in N$, $\varphi_i^{nt}(\gamma) = (0,0)$.[11]

Example 2 (***Individual Rationality, Non-Imposition, Pareto Efficiency***)
Fix a positive price $P > 0$. For each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, we assign the object to the agent with the lowest index, agent $\min N$, at fixed price $P > 0$, i.e.,

$$\varphi_i^1(\gamma) = \begin{cases} (1,P) & \text{if } i = \min N \text{ and} \\ (0,0) & \text{otherwise.} \end{cases}$$

Mechanism φ^1 does neither satisfy *individual rationality* (IR2) nor *non-imposition*, e.g., for problem $\gamma' = (N,u)$ with $1 \in N$ and u_1 such that $v_1 = 0$, we have $\alpha_1^1(\gamma') = 1$ and $\pi_1^1(\gamma') = P > 0 = v_1$. However, φ^1 satisfies *individual rationality* (IR1).

Alternatively, for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, we assign the object at price $P > 0$ to the agent with the lowest index within the set of agents who have a valuation larger than P. In addition, if agent 1 is present, he pays price P. For $N \in \mathcal{N}$ and $\gamma = (N,u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$ denote the set of agents who have a valuation larger than P by $P(\gamma) = \{i \in N : v_i > P\}$. Then,

$$\varphi_i^2(\gamma) = \begin{cases} (1,P) & \text{if } i = \min P(\gamma), \\ (0,P) & \text{if } 1 \in N \text{ and } 1 \neq \min P(\gamma), \text{ and} \\ (0,0) & \text{otherwise.} \end{cases}$$

Mechanism φ_i^2 does neither satisfy *individual rationality* (IR1) nor *non-imposition*, e.g., for problem $\gamma' = (N,u)$ with $1 \in N$ and u_1 such that $v_1 = 0$, we have $\alpha_1^2(\gamma') = 0$ and $\pi_1^2(\gamma') = P > 0$. However, φ^1 satisfies *individual rationality* (IR2).

Example 3 (***Strategy-Proofness***)
For each $N \in \mathcal{N}$ and each $\gamma = (N,u) \in \Gamma^N$ with associated valuation vector $v \in \mathcal{V}(N)$ we assign the object to the agent with the lowest index within the set of agents who have a positive valuation. Denote the set of agents who have a positive valuation by $O(\gamma) = \{i \in N : v_i > 0\}$.

[11] This mechanism was earlier used by Klaus and Miyagawa (2002, Example 3) and Sakai (2013).

The agent who obtains the object pays half his valuation.

$$\varphi_i^v(\gamma) = \begin{cases} (1, \frac{v_i}{2}) & \text{if } i = \min O(\gamma) \text{ and} \\ (0,0) & \text{otherwise.} \end{cases}$$

Example 4 (***Consistency***)
Let $f \in \mathcal{F}$ be a vector of reservation prices and $\succ, \succ' \in \Omega$ be two distinct priority orderings. We apply $\psi^{(f,\succ)}$ to problems where the set of agents has cardinality 2 and $\psi^{(f,\succ')}$ otherwise, i.e.,

$$\varphi_i^w(\gamma) = \begin{cases} \psi_i^{(f,\succ)}(\gamma) & \text{if } |N| = 2, \text{ and} \\ \psi_i^{(f,\succ')}(\gamma) & \text{otherwise.} \end{cases}$$

Example 5 (***Non Wasteful Tie-Breaking, Pareto Efficiency***)
Consider a modification of our serial dictatorship mechanism with reservation prices in which agents who are indifferent between [not receiving the object and not paying anything] and [receiving the object and paying his reservation price], always take the object. That is, given $f \in \mathcal{F}$ and $\succ \in \Omega$, for each $N \in \mathcal{N}$ and each $\gamma \in \Gamma^N$, a modified serial dictatorship $\varphi^{mod\ (f,\succ)}$ assigns a uniquely determined outcome $\varphi^{mod\ (f,\succ)} \in \mathcal{O}(N)$ just like the serial dictatorship $\psi^{(f,\succ)}$ would, but using $U^{mod}(\gamma) \equiv \{i \in N : v_i \geq f_i\}$ instead of $U^f(\gamma) = \{i \in N : v_i > f_i\}$.

8 Conclusion

In a simple setup where one indivisible object is to be allocated to exactly one among many agents, we proposed a set of normative criteria, and we introduced a new *serial dictatorship with reservation prices* mechanism that combines priorities over agents and individual reservation prices. Our main result shows that a mechanism φ satisfies *minimal tradability*, *individual rationality*, *strategy-proofness*, *consistency*, and *non wasteful tie-breaking*, if and only if there exists a reservation price vector r and a priority ordering $\succ$ such that φ is a serial dictatorship mechanism with reservation prices based on r and $\succ$.

Apart from providing a theoretical foundation for serial dictatorship mechanisms (with and without reservation prices) we can provide some insights about how some allocation mechanisms work in various real markets, from the allocation of the next-available consultant-led medical appointment, to on-board flight upgrading, to the prioritization of traffic in a computer network, and so on. For concreteness, we focus on and detail one specific example, the allocation of the next-available consultant-led medical appointment in Australia.

Under Commonwealth federal law, residents in Australia are covered by Medicare universal health insurance, which provides free or subsidized health care services. Private insurance is optional, subscribed to by roughly one in two, and is generally used as a top-up to Medicare providing additional benefits.[12] For instance, it may reduce or eliminate out-of-pocket costs (also known as "gap payments"). Due to the large number of insurance options and personal circumstances, even for the exact same health care procedure or service, out-of-pocket costs are idiosyncratic.[13]

State and territory governments administer certain elements of health care within their jurisdiction, such as the operation of public hospitals, through charters that set the regulatory framework for the within state provision of medical services. While expressed in plain language, these charters include many requirements that are essentially normative criteria that the service providers should comply with. For example, in the state of Victoria, clinical prioritization requires "equality of access to specialist clinic services" and that specialist clinic appointments are "actively managed to ensure patients are treated equitably within clinically appropriate timeframes and with priority given to patients with an urgent clinical need."[14] In practice, the public hospitals implement these requirements by creating a priority order for specialist services induced by clinical need and arrival time, but which ignores the patients' insurance status. Given the prevailing priority order, the next-available specialist service appointment is offered to the patient with the highest priority, who considers his value and out-of-pocket cost for it, and chooses whether to accept it or not. If the patient accepts, the appointment is allocated to him. Otherwise, the appointment is offered to the patient with the next highest priority, who chooses whether to accept it or not, and so on. All appointments made are nominal and patients cannot trade appointments among themselves.[15]

The *serial dictatorship mechanism with reservation prices* mimics the procedure that the hospitals arrived at for allocating the next-available appointment, where the hospital's

[12]Private insurance is compulsory for anyone who is not an Australian citizen or a permanent resident.

[13]There are many private insurers and each typically offers many policies that differ on the basis of the range of treatments that are covered, to what extent those treatments are covered, waiting periods, level of excess or co-payments that one may be required to pay, the price and discounts available, and so on.

[14]See the "Access Policy" white paper by the Health Service Programs Branch (2013).

[15]While our description above applies in many situations, there are several exceptions and limitations. For instance, for organ transplants appointments are made using a dynamic matching procedure that weights waiting for more available organs in order to create a larger compatibility pool against greedily matching and assigning the next-available appointment (Akbarpour et al., 2016). Emergency room rules for dealing with life threatening situations are also different. More generally, people in very serious conditions are unlikely to pass their turn. Our description is best suited for procedures that are less severe, but nevertheless serious enough to require a specialist-led appointment, including for instance most "watchful waiting" scenarios in which the condition does require a specialist-led appointment that the patients then decide whether or not to take.

priority order for specialist services is taken as an exogenously given priority ordering[16] and the out-of-pocket costs to be paid by the patients are interpreted as their reservation price for the service. The patient with the highest priority is considered first. If his value for the next-available appointment is strictly higher than his reservation price, then he obtains this appointment, all remaining patients receive and pay nothing, and we stop. Otherwise, he receives and pays nothing, and we continue.

At the same time, our normative criteria capture some of the requirements set by the states in the charter that specifies the regulatory framework as follows. *Minimal tradability* asks that for any group of patients, there exists a utility profile such that one of them can obtain the appointment. *Individual rationality* specifies that a patient who does not receive an appointment pays nothing, whereas if he receives it, the out-of-pocked amount that he pays cannot exceed his valuation for it.[17] *Strategy-proofness* asks that no patient can profitably misreport his true utility for the appointment; it avoids outcomes that are based on strategic manipulations and levels the playing field, ensuring "equality of access", in the sense that "sophisticated" patients cannot get an edge over "unsophisticated" ones by misreporting their utility. *Consistency* requires that if some patients who did not receive the appointment withdraw from the queue (possibly because they no longer need it), the outcome for everyone else remains unchanged. *Non wasteful tie-breaking* excludes that the appointment is allocated to a patient at some price if he is indifferent between this outcome and [not receiving an appointment and not paying anything]. That is, the mechanism does not "waste" the appointment on a patient who is indifferent since another patient might strictly prefer to receive it.

As a first order approximation, our normative criteria seem to capture reasonably well the requirements set by the states in the charters for the provision of medical services, and our serial dictatorship mechanism with reservation prices mimics the procedure that the hospitals arrived at for the allocation of the next-available specialist-based appointment. In this context, our main characterization result indicates that the current procedure for allocating medical appointments via a serial dictatorship with externally determined reservation prices and priorities is aligned with current public health care guidelines; moreover, this is the only procedure that respects those normative criteria.

[16]Clinical need is most of the time established based on external referrals, the arrival time is random, and hospitals use consistent procedures to determine the priorities. Thus, although the priority order is created within the hospital, it can be interpreted as being essentially exogenously given.

[17]*Non-imposition* allows patients who are not interested in the appointment to withdraw from consideration at no cost.

References

Akbarpour, M., S. Li, and S. O. Gharan (2016). Thickness and information in dynamic matching markets. *Working paper.*

Bade, S. (2015). Serial dictatorship: the unique optimal allocation rule when information is endogenous. *Theoretical Economics 10*(2), 385–410.

Chakraborty, A., A. Citanna, and M. Ostrovsky (2015). Group stability in matching with interdependent values. *Review of Economic Design 19*(1), 3–24.

Dold, M. and M. Khadjavi (2017). Jumping the queue: An experiment on procedural preferences. *Games and Economic Behavior 102*, 127–137.

Doğan, B. and B. Klaus (2016). Object allocation via immediate-acceptance: Characterizations and an affirmative action application. Cahier de recherches économiques du DEEP No. 16.15.

Ehlers, L. and B. Klaus (2003). Coalitional strategy-proof and resource-monotonic solutions for multiple assignment problems. *Social Choice and Welfare 21*(2), 265–280.

Ehlers, L. and B. Klaus (2007). Consistent house allocation. *Economic Theory 30*(3), 561–574.

Ehlers, L. and B. Klaus (2014). Strategy-proofness makes the difference: Deferred-acceptance with responsive priorities. *Mathematics of Operations Research 39*(4), 949–966.

Ehlers, L. and B. Klaus (2016). Object allocation via deferred-acceptance: Strategy-proofness and comparative statics. *Games and Economic Behavior 97*, 128–146.

Ehlers, L., B. Klaus, and S. Pápai (2002). Strategy-proofness and population-monotonicity for house allocation problems. *Journal of Mathematical Economics 38*(3), 329–339.

Ergin, H. (2000). Consistency in house allocation problems. *Journal of Mathematical Economics 34*(1), 77–97.

Hatfield, J. W. (2009). Strategy-proof, efficient, and nonbossy quota allocations. *Social Choice and Welfare 33*(3), 505–515.

Health Service Programs Branch, Victorian Government, D. o. H. (2013). *Specialist Clinics in Victorian Public Hospitals - Access Policy.*

Hylland, A. and R. Zeckhauser (1979). The efficient allocation of individuals to positions. *Journal of Political Economy 87*(2), 293–314.

Klaus, B. (2001a). Coalitional strategy-proofness in economies with single-dipped preferences and the assignment of an indivisible object. *Games and Economic Behavior 34*(1), 64–82.

Klaus, B. (2001b). Population-monotonicity and separability for economies with single-dipped preferences and the assignment of an indivisible object. *Economic Theory 17*(3), 675–692.

Klaus, B. and E. Miyagawa (2002). Strategy-proofness, solidarity, and consistency for multiple assignment problems. *International Journal of Game Theory 30*(3), 421–435.

Klaus, B., H. Peters, and T. Storcken (1997). Strategy-proof division of a private good when preferences are single-dipped. *Economics Letters 55*(3), 339–346.

Kojima, F. and M. Manea (2010). Axioms for deferred acceptance. *Econometrica 78*(2), 633–653.

Kojima, F. and U. Ünver (2014). The "Boston" school-choice mechanism: An axiomatic approach. *Economic Theory 55*(3), 515–544.

Konow, J. (2003). Which is the fairest one of all? a positive analysis of justice theories. *Journal of Economic Literature 41*(4), 1188–1239.

Li, S. (2016). Obviously strategy-proof mechanisms. *SIEPR Discussion Paper No. 16-015.*

Mann, L. (1969). Queue culture: The waiting line as a social system. *American Journal of Sociology 75*(3), 340–354.

Pápai, S. (2000). Strategyproof assignment by hierarchical exchange. *Econometrica 68*, 1403–1433.

Pápai, S. (2001). Strategyproof single unit award rules. *Social Choice and Welfare 18*(8), 785–798.

Sakai, T. (2008). Second price auctions on general preference domains: Two characterizations. *Economic Theory 37*(2), 347–356.

Sakai, T. (2013). Axiomatizations of second price auctions with a reserve price. *International Journal of Economic Theory 9*(3), 255–265.

Svensson, L.-G. (1999). Strategy-proof allocation of indivisible goods. *Social Choice and Welfare 16*(4), 557–567.

Thomson, W. (1983). The fair division of a fixed supply among a growing population. *Mathematics of Operations Research 8*(3), 319–326.

Thomson, W. (2015). *Consistent Allocation Rules.* Cambridge University Press. forthcoming.